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ENERGY DOUBLER/SAVER RADIOFREQUENCY ACCELERATION SYSTEM

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A. BASIC ASSUMPTIONS

The basic assumptions of the energy doubler/saver lattice characteristics are:

1. Orbit circumference = $\frac{1112}{1113}$ x (MR circumference)
2. Transition-γ = MR-γ_t = 18.75

The assumptions for the beam transfer and acceleration are:

1. The beam is synchronously transferred from the MR to the doubler, i.e., without debunching and rebunching which is always a lossy process. Hence, the doubler harmonic number should be integral multiples of 1112 and the sizes and shapes of the MR and doubler rf buckets should be properly matched at transfer.

2. The beam is transferred at 300 GeV where $1-\beta = 4.86 \times 10^{-6}$. To get fixed orbital frequency up to infinite energy the orbit length deviation must be $\frac{\Delta L}{L} = \frac{\Delta\beta}{\beta} = 4.86 \times 10^{-6}$ and the maximum orbit excursion at $x_{p\ max}$ is

$$(\Delta x)_{max} = \gamma_t^2 \ \frac{\Delta L}{L} x_{p\ max} = (1.71 \times 10^{-3})\ x_{p\ max} = 10.25 \text{ mm} \qquad (1)$$

for $x_{p\ max}$ = 6m. If the aperture is large enough to contain this orbit excursion the rf system can be operated at fixed frequency.

B. BUCKET MATCHING AND ACCELERATION RATE

For the longitudinal phase-space we use the physical coordinates Δz and Δp. The central part of a bucket is, then, given by the ellipse

$$\frac{(\Delta p)^2}{a^2} + \frac{(\Delta z)^2}{a^{-2}} = \frac{A}{\pi} \tag{2}$$

where A is the area of the ellipse and

$$a^4 = \frac{h}{2\pi}\left(\frac{mc}{R}\right)^2\left(\frac{eV\cos\phi_s}{mc^2}\right)\frac{\gamma}{\Lambda} \tag{3}$$

with $\Lambda = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2} = 2.8347 \times 10^{-3}$.

(Since we are always above transition it is convenient to measure the synchronous phase ϕ_s backward from the downward crossing.) To match the shapes of the MR (subscript M) and the doubler (subscript D) buckets we must have $a_D = a_M$ or

$$\left(\frac{hV\cos\phi_s}{R^2}\right)_D \doteq \left(\frac{hV\cos\phi_s}{R^2}\right)_M \tag{4}$$

or

$$\frac{(V\cos\phi_s)_D}{(V\cos\phi_s)_M} = \frac{h_M}{h_D}\left(\frac{R_D}{R_M}\right)^2 = \frac{1112}{h_D} \cdot \frac{1112}{1113} \tag{5}$$

The bucket area in these physical coordinates is

$$A_b = 8\ mcR\left(\frac{2}{\pi h^3}\frac{eV}{mc^2}\frac{\gamma}{\Lambda}\right)^{\frac{1}{2}}\alpha(\phi_s) \tag{6}$$

where $\alpha(\phi_s)$ is the ratio of bucket areas at $\phi_s=\phi_s$ and at $\phi_s=0$, and is tabulated in CERN/MPS-SI/Int. DL/70/4 (23 April, 1970). The measured beam bunch area in the MR (at 300 GeV) is $\sim$ 0.1 eV sec. <u>Hence, we require that A_b > 0.2 eV sec so that the beam bunch is approximately elliptical and proper matching can be achieved.</u>

With a chosen h_D the other doubler rf parameters are totally determined by $(V \cos \phi_s)_M$ and A_{bD} through Equs. (5) and (6). For

$$(V \cos \phi_s)_M = 3.5 \text{ MV}, \quad A_{bD} = 0.2 \text{ eV sec} \tag{7}$$

and various h_D we get

TABLE 1

h_D	$V_D \cos \phi_{sD}$ (MV)	ϕ_{sD}	$\alpha(\phi_{sD})$	V_D (MV)	$V_D \sin \phi_{sD}$ (MV)	$\dot{E}_D$ (GeV/sec)
1112	3.497	75.96°	0.008913	14.412	13.982	667.7
2224	1.748	62.09°	0.04952	3.735	3.300	157.6
3336	1.166	48.57°	0.13247	1.761	1.321	63.07
4448	0.874	35.60°	0.26106	1.075	0.626	29.89
5560	0.699	23.41°	0.43335	0.7621	0.303	14.46
6672	0.583	12.32°	0.64385	0.5966	0.127	6.08

C. DISCUSSION

For h_D = 1112 and 2224 the doubler rf voltage V_D and acceleration rate $\dot{E}_D$ given in Table 1 are unnecessarily large because of the assumed value of 3.5 MV for $V_M \cos \phi_{sM}$. This makes the beam bunch in the MR rather high (Δp) compared to its width (Δz), thereby requiring a large V_D for shape matching. If one wants only $\dot{E}_D$ = 100 GeV/sec or $V_D \sin \phi_{sD}$ = 2.094 MV (beam revolution time in doubler = 20.94 μsec) the conditions instead of Equ. (7) should rather be

$$V_D \sin \phi_{sD} = 2.094 \text{ MV}, \quad A_{bD} = 0.2 \text{ eV sec} \tag{8}$$

Then for h_D = 1112 and 2224 we have

TABLE 2

h_D	ϕ_{sD}	$\alpha(\phi_{sD})$	V_D (MV)	$V_M \cos \phi_{sM}$ (MV)
1112	69.61°	0.02264	2.234	0.7789
2224	59.58°	0.06141	2.428	2.4616

The simplest way to obtain these values of $V_M \cos \phi_{sM}$ is to flattop the MR (ϕ_{sM} = 0) and, then, turn the rf voltage down to the values given in Table 2. The beam bunches will be matched to the doubler bucket with rf voltage V_D given in Table 2 and the magnet ramping at 100 GeV/sec.

For h_D > 2224 the doubler acceleration rate is too low. To increase $\dot{E}_D$ one must either increase $V_M \cos \phi_{sM}$ or decrease A_{bD}. Neither alternative is feasible. Therefore we must either start $\dot{E}_D$ at transfer at the values given, then turn up the rf voltage and the magnet ramp-rate adiabatically afterwards or we can give up bucket-shape matching and tolerate a dilution in the longitudinal phase-space. In the latter case we should keep $V_M \cos \phi_{sM}$ at its maximum value of 3.5 MV to minimize dilution and keep the values of ϕ_{sD} given in Table 1 which are maximum values to still provide adequate bucket width. For $V_D \sin\phi_{sD} = 2.094$ MV and for various values of h_D we then have

TABLE 3

h_D	ϕ_{sD}	V_D (MV)	dilution
3336	48.57°	2.793	79.42%
4448	35.60°	3.597	54.67%
5560	23.41°	5.271	38.02%
6672	12.32°	9.812	24.66%

Where the dilution factor is simply given by

$$\text{dilution} = \frac{\text{diluted density}}{\text{initial density}} = \left[\frac{V_D(\text{Table 1})}{V_D(\text{Table 3})}\right]^{\frac{1}{2}}$$

We could perhaps reduce ϕ_{sD} from values given in Table 1 and obtain better matching, hence less dilution. But this would require larger V_D than the already large values given in Table 3.

For example, to operate at $h_D = 4448$ (doubler rf frequency = 212.42 MHz), with the doubler magnet ramped at 100 GeV/sec the rf voltage should be $V_D = 3.597$ MV giving a synchronous phase of $\phi_{sD} = 35.60^\circ$. The width (Δz) of the beam bunch will be unchanged but the height (Δp) will be increased by $(\text{dilution})^{-1} = 1.829$ times.